



TransINT: Embedding Implication Rules in Knowledge Graphs with Isomorphic Intersections of Linear Subspaces

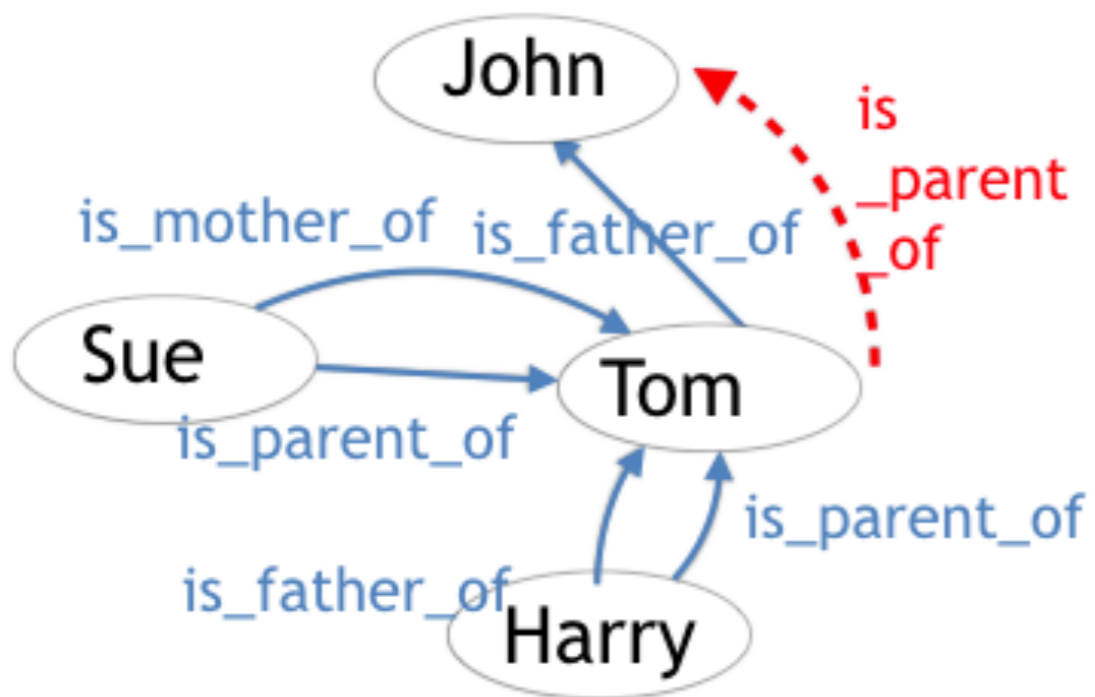
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Knowledge Graph Embeddings



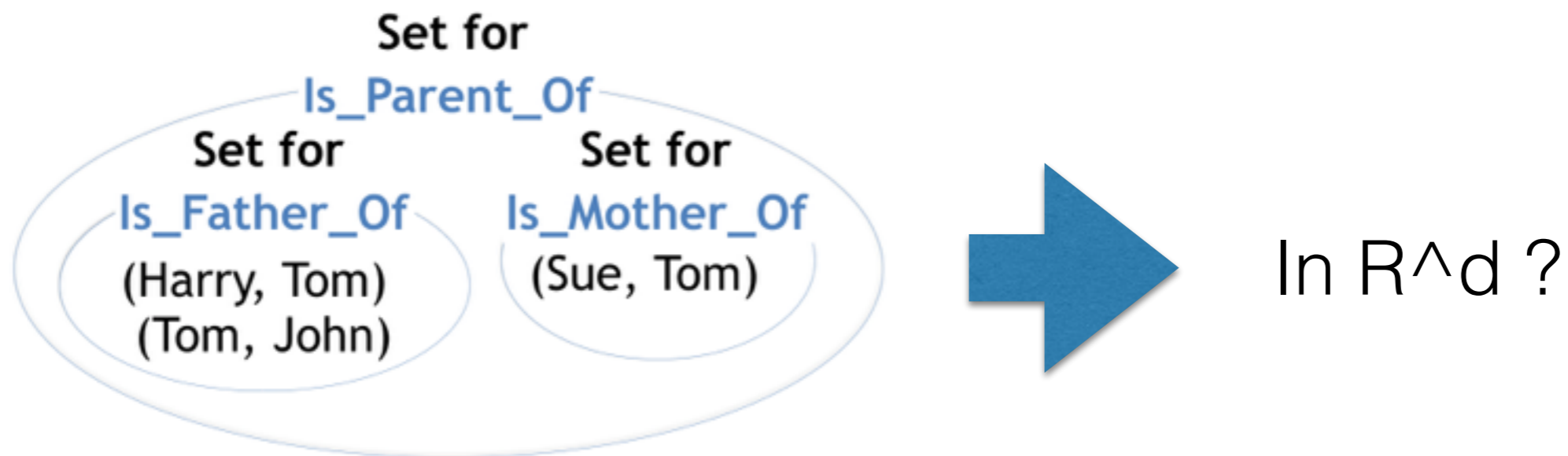
▲ A Knowledge Graph

Ex) Implication Rule:
Is_Father_Of -> Is_Parent_Of

Can we impose
Implication Rule in KG Embedding?

Problem Statement

- Relation as sets of ordered pairs of entities
- Given rules or hierarchy



Knowledge Graph

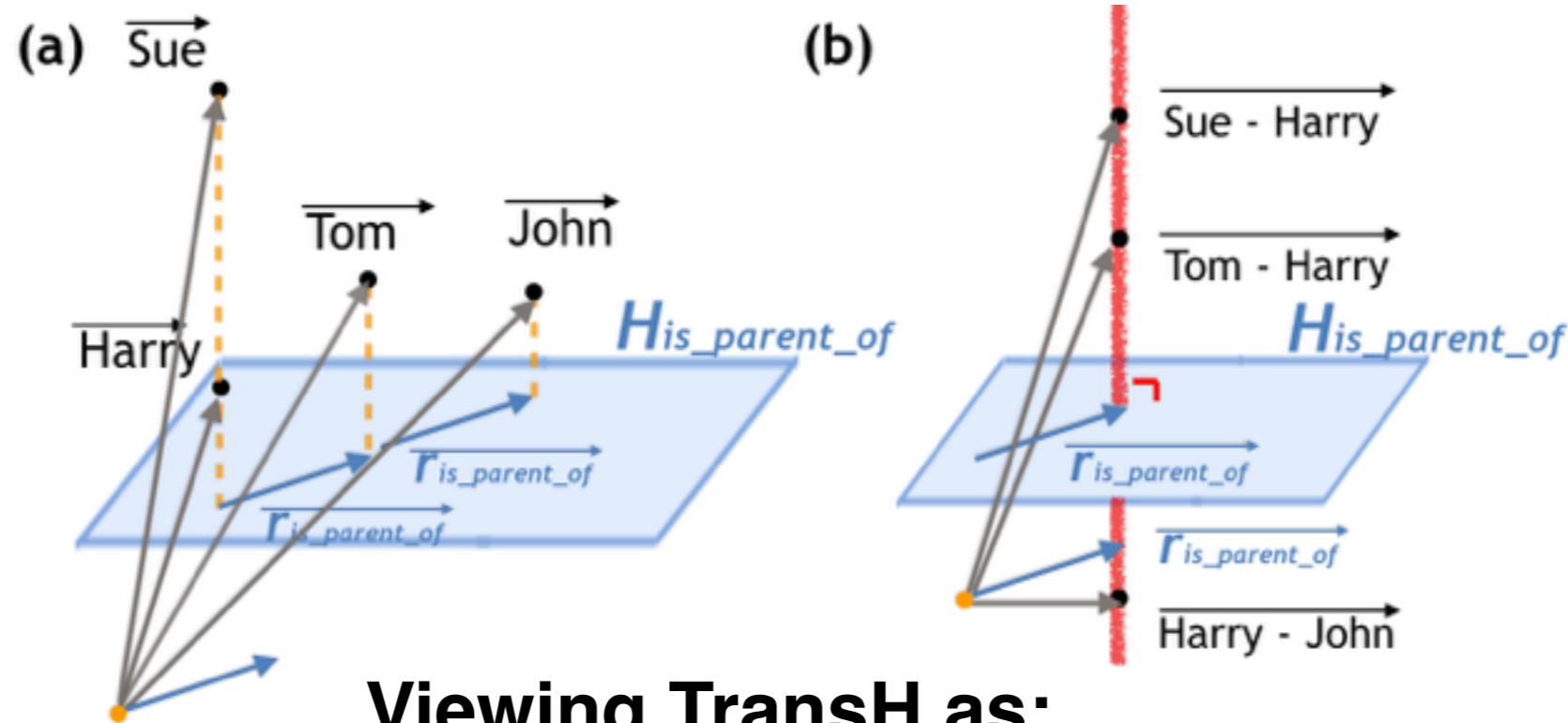
Embeddings in R^d

Goal: $(R_i \subset R_j)$ iff $(r_i$'s relation space $\subset r_j$'s relation space)
or equivalently,
 $(r_i \Rightarrow r_j)$ iff $(r_i$'s relation space $\subset r_j$'s relation space)

TransH

Given $(h, r, t) \rightarrow \vec{h}, \vec{t}, \vec{r}, H_r$

A Different Perspective on TransH



Viewing TransH as:

(a): $\vec{t}_{\perp} - \vec{h}_{\perp} \approx \vec{r}$

(projection first, then difference)

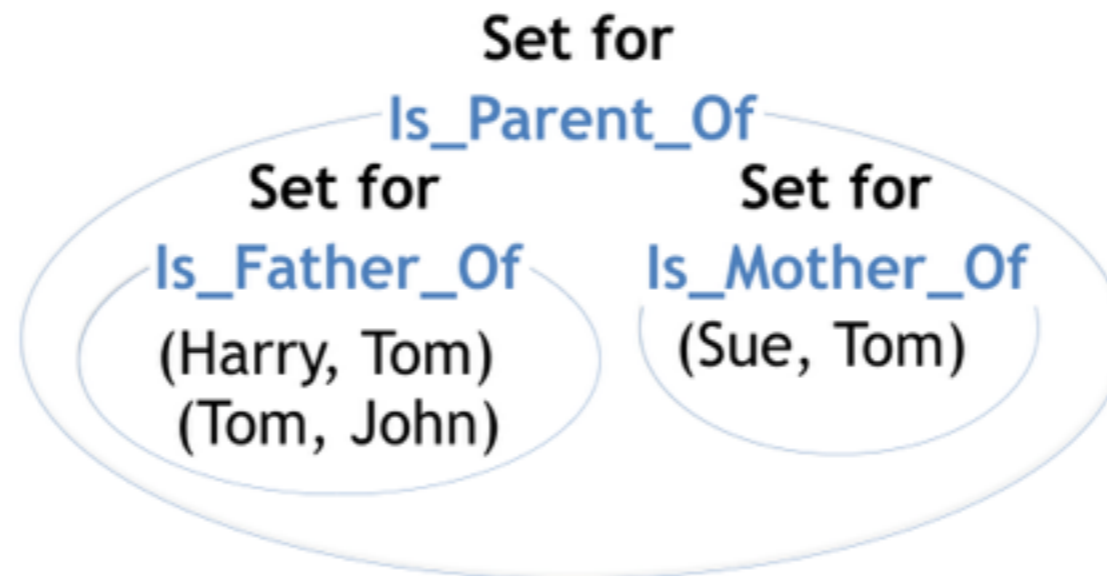
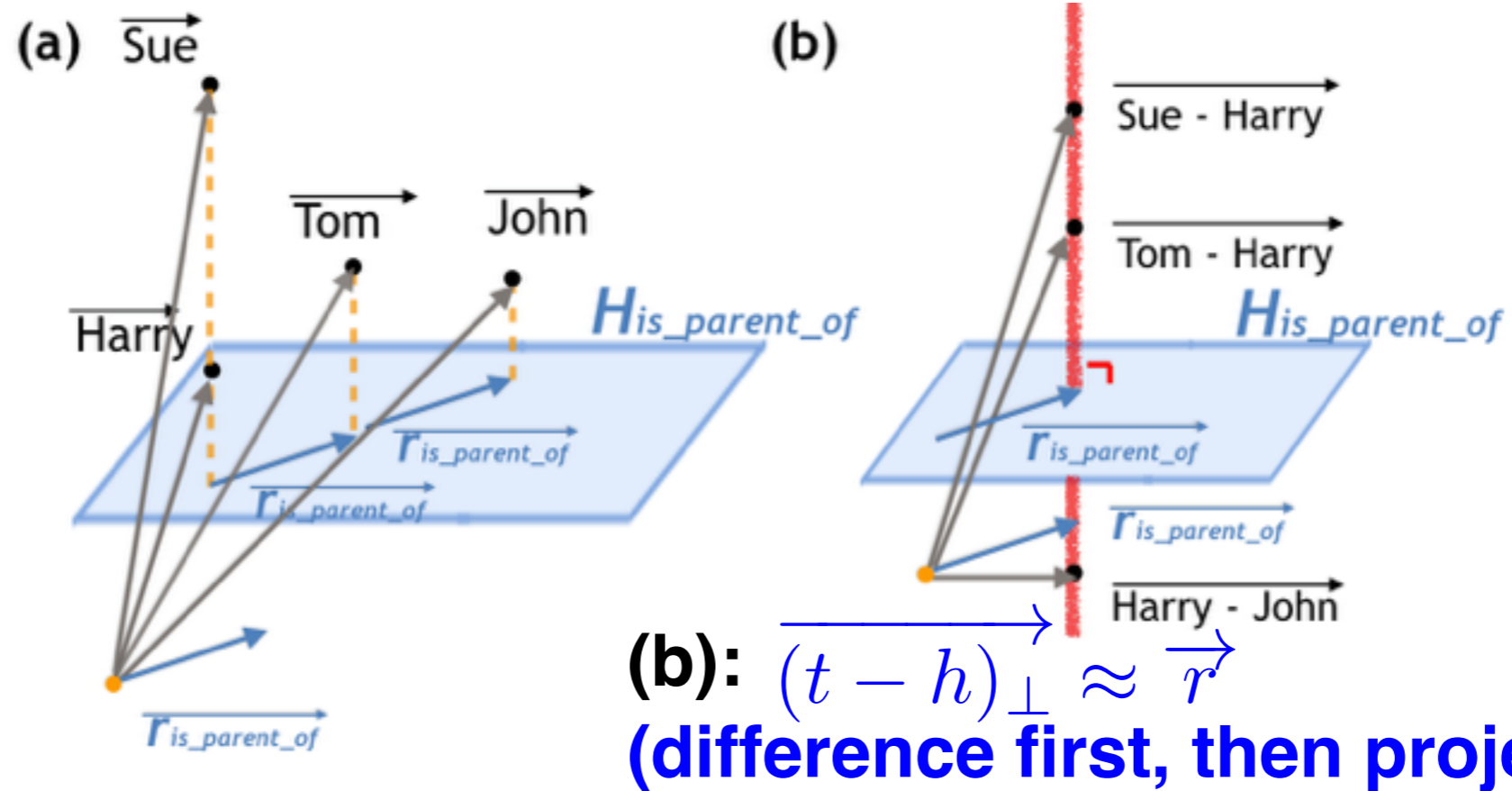
(b): $\overrightarrow{(t - h)}_{\perp} \approx \vec{r}$

(difference first, then projection)

Red Line: The space where $\overrightarrow{t - h}$ of all (h,t) 's tied by *is_parent_of* can exist.

= **Relation Space** of *is_parent_of*

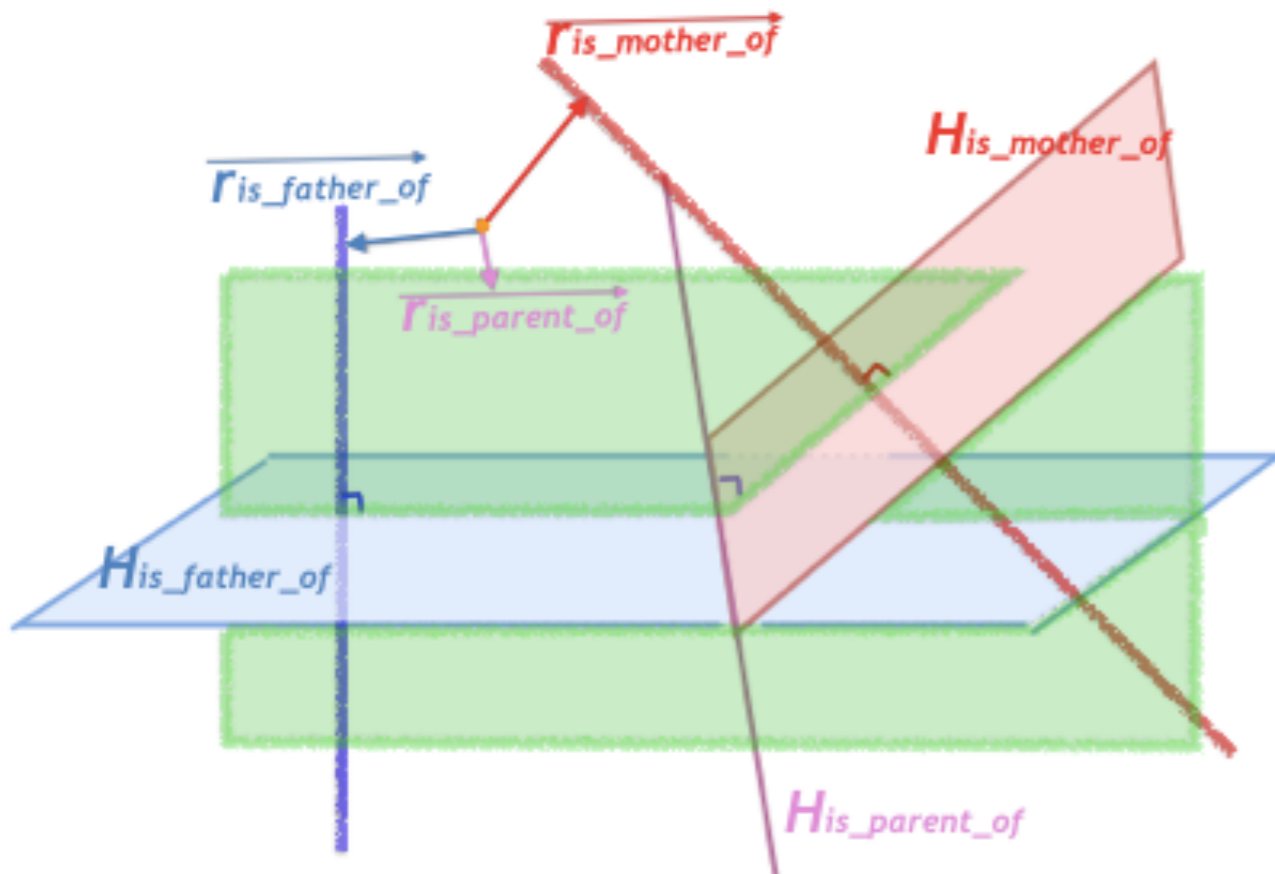
Methods



Methods

1. **Intersection Constraint:** $H_i \subset H_j$
2. **Projection Constraint:** Projection of \vec{r}_j onto H_i is \vec{r}_i

Or equivalently
 $P_i \vec{r}_j = \vec{r}_i$ where P_i is projection matrix onto H_i .



$\text{rel.space}(is_father_of),$
 $\text{rel.space}(is_mother_of)$
 $\subset \text{rel.space}(is_parent_of)$

Green hyperplane = relation space of is_parent_of

Blue, red lines: relation space of $is_father_of, is_mother_of$

Results - Link Prediction

Link Prediction: predict ? in (h, r, ?)

ex) (Harry, is_father_of, ?)

Table 1: Results for Link Prediction on FB122. *: For KALE, we report the best performance by any of KALE-PRE, KALE-Joint, KALE-TRIP (3 variants of KALE proposed by Guo et al. [2016]).

	Raw					Filtered				
	MRR	MED	Hits N%			MRR	MED	Hits N%		
			3	5	10			3	5	10
TransE	0.262	10.0	33.6	42.5	50.0	0.480	2.0	58.9	64.2	70.2
TransH	0.249	12.0	31.9	40.7	48.6	0.460	3.0	53.7	59.1	66.0
TransR	0.261	15.0	28.9	37.4	45.9	0.523	2.0	59.9	65.2	71.8
KALE*	0.294	9.0	36.9	44.8	51.9	0.523	2.0	61.7	66.4	72.8
TransINT^G	0.339	6.0	40.1	49.1	54.6	0.655	1.0	70.4	75.1	78.7
TransINT^{NG}	0.323	8.0	38.3	46.6	53.8	0.620	1.0	70.1	74.1	78.3

Table 2: Results for Link Prediction on NELL sport/ location.

	Sport					Location				
	MRR		Hits N%			MRR		Hits N%		
	Filtered	Raw	1	3	10	Filtered	Raw	1	3	10
Logical Inference	-	-	28.8	-	-	-	-	27.0	-	-
Simple	0.230	0.174	18.4	23.4	32.4	0.190	0.189	13.0	21.0	31.5
Simple+	0.404	0.337	33.9	44.0	50.8	0.440	0.434	43.0	44.0	45.0
TransINT^G	0.450	0.361	37.6	50.2	56.2	0.550	0.535	51.2	56.8	61.1
TransINT^{NG}	0.431	0.362	36.7	48.7	52.1	0.536	0.534	51.1	53.3	59.0

Results - Triple Classification

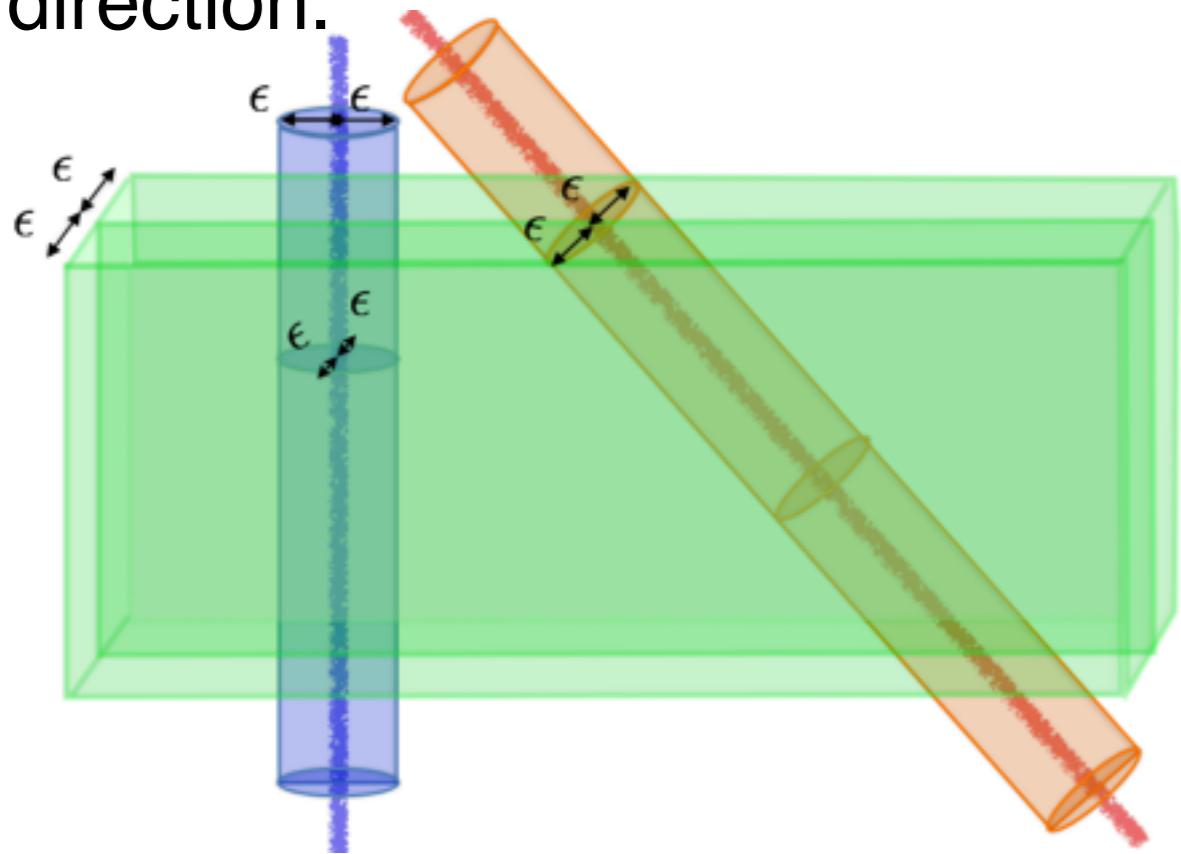
Triple Classification: predict (h, r, t) valid/ invalid

Table 3: Results for Triple Classification on FB122, in Mean Average Precision (MAP).

TransE	TransH	TransR	KALE*	TransINT^G	TransINT^{NG}
0.634	0.641	0.619	0.677	0.781 (0.839/ 0.752)	0.743 (0.709/ 0.761)

Margin-aware geometry

Margin-aware Geometry (Fig. (d)): If weaker objective required (i.e. $(t - h)_{\perp} - \vec{r} < \epsilon$), relation spaces (now with thickness 2ϵ) still included in the same direction.

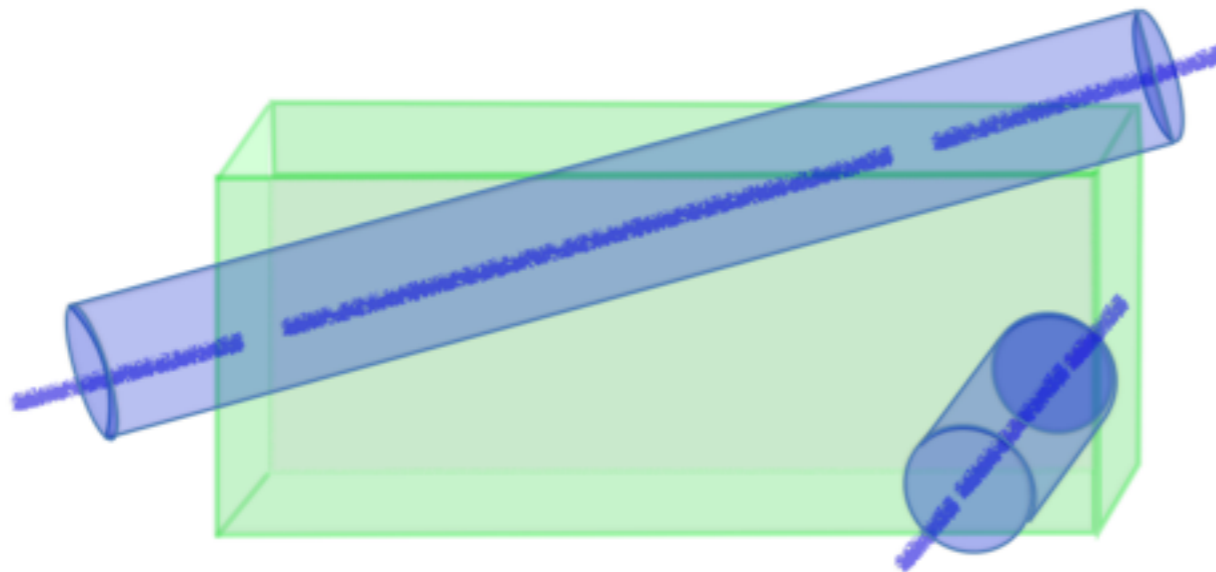


$\text{rel.space}(is_father_of),$
 $\text{rel.space}(is_mother_of)$
 $\subset \text{rel.space}(is_parent_of)$

Green hyperrectangle = rel. space of *is_parent_of*

Blue, red cylinders: rel. space of *is_father_of*, *is_mother_of*

Semantic Relatedness/ Rule Mining



▲ Relation Spaces overlapping with different angles

Angle:

Metric of similarity between embedded objects

Ex) Word2vec, Visual Semantics

Between two relation spaces, the closer the angle



the more overlap in area

Table 4: Examples of relations' angles and *imb* with respect to `/people/person/place_of_birth`

		Relation	Angle	<i>imb</i>
Not Disjoint	Relatedness	<code>/people/person/nationality</code>	22.7	1.18
	Implication	<code>/people/person/place_lived/location*</code>	46.7	3.77
Disjoint		<code>/people/cause_of_death/people</code>	76.6	n/a
		<code>/sports/sports_team/colors</code>	83.5	n/a